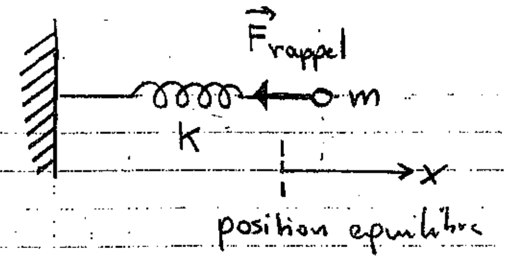
I Mouvement périodique d'une masse = oscillateur



$$\sum \vec{F} = m\vec{a}$$

$$\underbrace{\vec{P} + \vec{R}}_{\vec{0}} + \vec{F}_{rappel} = m\vec{a} \quad \text{sans frottement}$$

$\longrightarrow$ Pb à 1 dimension

$$m\vec{a} = \vec{F}_{rappel} \longrightarrow m \frac{d^2x(t)}{dt^2}\vec{e_x} = -k\,x\,\vec{e_x}$$

I.1 $\quad \frac{d^2x(t)}{dt} + \frac{k}{m}x(t) = 0 \qquad \frac{d^2x(t)}{dt^2} + \omega_0^2 x(t) = 0 \qquad \boxed{\omega_0^2 = \frac{k}{m}}$

A.N. $\quad m = 0{,}1\ kg \quad K = 10\ kg.s^{-2} \quad \omega_0 = \sqrt{\frac{10}{0{,}1}} = 10\ rad.s^{-1}$

I.2 éq. caractéristique $e^{rt} \rightarrow r^2 + \omega_0^2 = 0 \qquad r = \pm i\omega_0$

$x(t) = A\,e^{i\omega_0 t} + B\,e^{-i\omega_0 t}$ combinaison linéaire

I.3.

$x_1(t) = A_1 \cos\omega_0 t + B_1 \sin\omega_0 t = \frac{A_1 - iB_1}{2} e^{i\omega_0 t} + \frac{A_1 + iB_1}{2} e^{-i\omega_0 t}$ $\quad$ m̂ forme $x(t)$

$x_2(t) = A_2 \cos(\omega_0 t + \phi_2) = \underbrace{A_2 \cos\phi_2}_{cte} \cos\omega_0 t - \underbrace{A_2 \sin\phi_2}_{cte} \sin\omega_0 t$

m̂ forme que $x_1(t) \rightarrow$ m̂ forme que $x(t)$

$x_3(t) = A_3 \sin(\omega_0 t + \phi_3) = A_3 \sin\phi_3 \cos\omega_0 t + A_3 \cos\phi_3 \sin\omega_0 t$

m̂ forme que $x_1(t) \rightarrow$ m̂ forme que $x(t)$

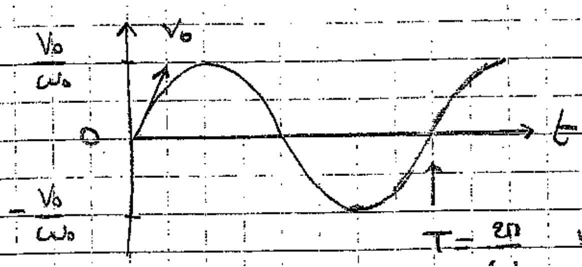
I.4 $\quad x(t=0) = 0 \quad \frac{dx}{dt}(t=0) = V_0$

$x(t=0) = A + B = 0 \qquad A = -B$

$\frac{dx}{dt}(t=0) = i\omega_0(A - B) = V_0 \longrightarrow A = \frac{V_0}{2i\omega_0}$

$$\boxed{x(t) = \frac{V_0}{\omega_0}\sin\omega_0 t}$$

Remarque $x_1(t) \rightarrow A_1 = 0 \quad B_1 = \frac{V_0}{\omega}$, $x_2(t) \rightarrow A_2 = \frac{V_0}{\omega_0} \quad \phi_2 = \frac{3\pi}{2}$, $x_3(t) \rightarrow A_3 = \frac{V_0}{\omega} \quad \phi_3 = 0$



$T = \frac{2\pi}{\omega}$ même période